From Sampled-data Control to Signal Processing
Part B

Yutaka Yamamoto
And
Masaaki Nagahara

Thanks to
Pramod Khargonekar,
U. Florida

and also to Kengo Zenitani, Shi Hongie, Kosuke Oku,

Message of Part B

- Many problems in the current signal processing paradigm
- Sampled-data control theory can allow better signal processing

Agenda for Part B

- Overview of the various difficulties of the band-limiting approaches
- Signal reconstruction problem via sampled-data control and its solution
- Comparison with non-ideal sampling theory (Unser, Elder, etc.)
- Applications
  - Audio and image processing
  - Super resolution
  - Voice restoration
  - Fractional delay filters and sound synthesis
  - Etc.
Overview of the current difficulties

In 1949 Claude Shannon wrote (now a classic) paper

- How fast should we sample in transmitting data through a channel?
- \( \Rightarrow \) Sampling theorem

Claude Elwood Shannon (1916-2001)

Message there

- Given a sampling period \( h \).
- There exists an upper bound of high frequency that one can transmit/store without a loss
- This frequency = \( \pi/h \) [rad/sec]= Nyquist frequency
- \( \Rightarrow \) One should limit the frequency contents below \( \pi/h \).

Typical difficulty

Sampling \( \rightarrow \) Aliasing

- High-freq. intersample information can be lost
- If no high-freq. components beyond the Nyquist frequency (=1/2 of sampling freq.) \( \rightarrow \) unique restoration
- \( \Rightarrow \) Whittaker-Shannon-Someya sampling theorem
Sampling Theorem

Band limiting hypothesis ⇒ unique recovery
\( \hat{f}(j\omega) = 0 \) for \( |\omega| > \frac{\pi}{h} \) ⇒

\[
f(t) = \sum_{n=-\infty}^{\infty} f(nh) \frac{\sin \pi(t/h - n)}{\pi(t/h - n)}
\]

Messages of this talk

- If we do not insist on perfect reconstruction, we can do better
- \( H^\infty \)-optimization
  - ⇒ yields less overall distortion
  - ⇒ optimal high-frequency reconstruction
  - ⇒ much less processing delays

Drawback of the Shannon and current techniques

- Lots of distortion (we’ll see later)
- Need more high freq. components
- The formula not causal (need infinitely many future sampled values)
- Very slow convergence
- Must approximate the ideal filter in practice
  - ⇒ Sharp cut-off characteristics
    - High degree of filters (and hence a long delay)

Current literature

- Unser, Blu, Aldroubi and collaborators (EPFL at Lausanne)
- Eldar (Technion)
- All based on projection methods (orthogonal or not) Cannot take care of \( V^\perp \) (\( V = \) constructed signal space)
- Essentially the same defects as Shannon paradigm
Typical problems due to the lack of high frequency

Effect of a band-limiting filter

Big amount of ringing due to the Gibbs phenomenon

Very metallic sounds of CDs

Also,... Mosquito Noise for images

A moving image demo

- Superresolution
- Left – bicubic extension
- Right – proposed
Challenges

- We want more high frequencies
- But how?
- Don’t rely on the sampling theorem!
- Can we go beyond it?
- How? ⇒ Signal Model and sampled-data control theory

More aggressive message for our community

- Control theory can give a better signal processing technique.
- After all, we control signals, one way or another.
- ⇒ More signal processing via control theory!

Outline of this talk

- Review of current digital signal processing
- Sampled-data control theory; review
- Signal processing via sampled-data control
- Applications to sounds and images

Current digital signal processing - Basics

=)
Sampling continuous-time signals

0 h 2h 3h 4h 5h 6h 7h

This does not produce a sound

0 h 2h 3h 4h 5h 6h 7h

Hold device is necessary

Simple 0-order hold

Old CD players

More recent players

Oversampling DA converter

Real signals: how much high. freq. are there

FFT of an Analog Record: very wide range

Real signals: how much high. freq. are there

FFT of an Analog Record: very wide range
**CD recording**

- **Freq. domain energy distribution**
- **Recorded signal**
- **Nyquist freq.**

Sampling frequency: 44.1kHz
Nyquist frequency: 22.05kHz
Alleged audible limit: 20kHz

We end up with a very unnatural freq. components

**In contrast:** Digital Recording (CD): sharp anti-aliasing filter
No signal beyond 20kHz

But you won’t be able to hear them anyway??

**Signal Reconstruction Problem**

Problem: Find $K[z]$ such that $\|T_{ew}\|_\infty < \gamma$
w: exogenous signal that drives $F(s)$
$F(s)$: analog signal generator
y: target analog signal to be reconstructed
$K[z]$: digital filter to be designed
$H_h$: hold-device
$P(s)$: analog buffer amplifier
$e^{-Ls}$: pure-delay that allows for processing time

Idea:
- If $\|T_{ew}\|_\infty$ is small, this allows a good recover of the original analog signal $y$ over all frequencies
- But there is a limit due to the sampling period $h$
- Hence we upsampling, and optimally interpolate

Upsampled Signal Reconstruction
Problem: Find $K[z]$ satisfying $\|T_{ew}\| < \gamma$
Sampled-data $H_\infty$ control problem
How to solve this?

- Recall the fast-sample/fast-hold approximation in Part A
- Of course, we can also resort to theoretical solutions a la Bamieh, Hara, Kabamba, and others.

Fast-sample/fast-hold Approximation

Background idea:

Utilizing analog characteristic

Freq. domain energy distribution

conventional

Imaging

new

Nyquist freq.
**Example of F(s)**

Sampling period $h=1$, delay step $m=2$

$$F(s) = \frac{1}{(0.70187s + 1)(7.0187s + 1)}$$

**Interpolator via the proposed method**

- Proposed Square wave resp.
- Virtually no ringing

**Response of the Johnston filter**

- Big amount of ringing due to the Gibbs phenomenon
**Comparison with other non-ideal sampling methods**

**Challenges for non band-limitedness**

- Unser’s method [Unser et al. 1994, 2005]
  - Consistency
  - Oblique projection
  - Exponential splines
- Eldar’s method [Eldar and Dvorkind, 2006]
  - Min-max projection
- Compressed sensing [Donoho, 2006]
  - Sparsity assumption
  - L1 optimization
- Unser’s method

- Shannon sampling theorem

\[ f(t) = \sum_{n=-\infty}^{\infty} f(nh)\phi(t - nh), \quad \phi = \text{sinc} \]

- It can be extended for more general function
  - \( \phi \) is piecewise polynomial (B-spline) [Unser et al. 1993]
  - \( \phi \) is piecewise exponential (exp. B-spline) [Unser & Blu 2005]
- other functions are found in wavelet theory
### Unser’s method

Also extended for more general setting  
[Unser et al. 1994, 2005]

If the output is re-injected the system, it yields exactly the same output.

### Problems in Unser’s reconstruction

- \( V(\phi_2) = s\sum(n\phi_2 - k) \) is not rich
- Reconstruction methods often ignore effects caused by signals that is not in the space \( V(\phi_2) \) (e.g. aliasing).
- Consistency forces the evaluation in \( V(\phi_2) \) thereby ignoring \( V(\phi_2)^\perp \)
- Signal reconstruction algorithms based on projection with signal subspace \( V \) are non-causal.

### Optimal Reconstruction as Oblique Projection in \( L^2 \)

If \( \phi_1 = \phi_2 \)

\[
L^2 \quad \text{error (aliasing)}
\]

\[
V(\phi) = \{\sum(n\phi(-n)) | n \in \ell^2 \} \subset L^2
\]

If \( \phi_1 \neq \phi_2 \)

\[
L^2 \quad \text{error}^2
\]

This implies that if \( f \in V(\phi_2) \) (band-limited) then the reconstruction is perfect.

### Comparison with exponential spline by Unser & Blu (2005)

- Sampling period \( h = \mathbb{R} \)
- Refilter \( F_a(s) = \frac{1}{s+1} \)
- Tostfilter \( P(s) = \frac{1}{(s+1)(s+2)} \)
- Frequency model for \( H^\infty \) design \( F(s) = \frac{1}{s+0.5} \)
- Delay parameter for \( H^\infty \) design \( m = \mathbb{R} \)
The optimal filter by Unser & Blu

- Unser’s filter
  \[ K(z) = \frac{z^3 - 0.7264z^2 + 0.1621z - 0.0111}{z(0.05725z^2 + 0.07827z + 0.006011)} \]
- \( K(z) \) has a pole outside the unit circle (\( z = -1.2859 \))
- The unstable transfer function is realized as a non-causal impulse response, which is then truncated.

Non-causal part of \( K(z) \)

Reconstruction of rectangular wave

- Sampled-data \( H^\infty \)

Reconstruction of rectangular wave

- Unser & Blu

Eldar’s method

- Min-max optimization
  \[ \min_{K} \max_{f \in L^2} \| f - P_V(\phi_2)f \| / \| f \|_{L^2} \]
- Consistency is satisfied.
- Worst-case design

\[ f(t) \xrightarrow{\phi_1} c_1(n) \xrightarrow{K(z)} c_2(n) \xrightarrow{\phi_2} \hat{f}(t) = \sum c_2(n)\phi_2(t-n) \]
**Problems in Eldar’s reconstruction**

- The same as Unser’s method
  - Non-causality
  - Large error if \( f \not\in V(\phi_2) \)

**Comparison by Numerical Example**

- **Parameters**
  - \( \phi_1 \): zero-order hold function \( \rightarrow \) average sampling
  - \( \phi_2 \): 2nd order B spline for Eldar’s method, up-sampling (by 25) + zero-order hold for SD
  - \( F(s) = \frac{1}{0.8s + 1}(0.5s + 1) \)
  - \( m = 4 \) (reconstruction delay)
  - \( h = 1 \) (sampling period)

\[
\begin{align*}
  f(t) &\xrightarrow{\phi_1} (c_1(n)) \xrightarrow{\text{sampling}} K(z) \xrightarrow{c_2(n)} (\phi_2) \xrightarrow{\text{hold (given)}} f(t) = \sum c_2(n)\phi_2(t - n)
\end{align*}
\]

**Reconstructed signal**

For \( f(t) = \sin \pi(t - 0.5) \)

Min-max [Eldar&Dvorkind 2006]

- Long reconstruction delay due to non-causality
  - Small errors
  - Short delay

**Reconstruction error**

Min-max [Eldar&Dvorkind 2006]

Sampled-data \( H^{\infty} \)
Comparison with other periods

\[ x(t) = \sin \frac{2\pi}{T} (t - 0.5) \]

Observation

- Why the difference?
- \( L^2 \) is too large (or rough) to be compared with the reconstruction results
- On the other hand, comparing the results just with \( V \) (reconstruction space) [e.g., consistency] cannot capture behavior for \( V^1 \)
- Making comparison with \( FL^2 \) is a reasonable choice \( \rightarrow \) Signal generator model!

Literature

- M. Unser and co-workers: lots of papers in Trans. SP
- Y. Eldar, Trans. SP
  - “Signal reconstruction via \( H^\infty \) sampled-data control,” Delta-sigma conversion, fractional delay filters
- Mirkin & Meinsma, Trans. SP

Applications
Applications - Sounds

- Sound processing beyond the CD range
- High-end audio
- Signal reconstruction from compressed audio (iPod, etc.) ASIC, iPod App
- Mobile phone restoration
- Sound synthesizer

Application - Images

- Superresolution
  - For still images
  - For moving images
- Noise reduction
  - Real-time processing

A sound demo

Red: Original (up to 22kHz)
Blue: downsampled to 11k, and then processed 4 times upsampled via YY filter
A new product on iPhone

- **AiSound for iPod/iPhone App**
  - Extend the freq. range from 16kHz to 20kHz
  - On Apple store
  - Sounds still better than Fantabit
  - US$5.00
Example in MD(mini disk) players

This “YY filter” is implemented in custom LSI sound chips by SANYO (now Panasonic), and being used in MP 3 players, mobile phones, voice recorders. The cumulative sales have exceeded 50 million chips.

Effect evaluation on compressed audio via PEAQ program
- Tested on 100 compressed music sources via PEAQ (Perceptual Evaluation of Audio Quality)
- PEAQ values:
  - 0…indistinguishable from CD
  - -1…distinguishable but does not bother the listener
  - -2…not disturbing
  - -3…disturbing
  - -4…very disturbing
- Note how YY improves the sound quality

http://en.wikipedia.org/wiki/PEAQ

Compression formats: MP3, AAC, WMA
Bitrates: 64kbps, 96kbps, 128kbps
Showing average values
Original Image (Baboon: zoomed)

Super-resolution by Lanczos filter

Super-resolution by total variation (TV) regulaziation

Super-resolution by YY
Comparison with Eldar’s method

Original

Eldar’s method

Results

<table>
<thead>
<tr>
<th></th>
<th>Two macaws*</th>
<th>Average**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>34.25 dB</td>
<td>29.59 dB</td>
</tr>
<tr>
<td>Min-max</td>
<td>33.78 dB</td>
<td>29.38 dB</td>
</tr>
</tbody>
</table>

* Measured in PSNR (Peak Signal to Noise Ratio)
** Average of results of 24 images from Kodak Lossless True Color Image Suite

A moving image demo

- Left: Original → ↓ 2 → ↑ 2 → bicubic filter
- Right: Original → ↓ 2 → ↑ 2 → filter via sampled-data control theory
- What makes the difference: $H^\infty$ control theory for sampled-data systems

Expand this image to HD quality

Thanks to Kengo Zenitani, a super programmer.
Extension via Bicubic filter

Extension via Lanczos filter

Extension via YY filter + lcm algorithm

Voice Restoration in Mobile Phones
Application to mobile phone voices

- Various difficulties not encountered in music
- Very low sampling rate – distortion quite audible
- Must imitate the harmonic structure of our voice generator
- Must distinguish sonants (voiced) and voiceless sounds
- Extension of the frequency range (YY)

Restoration from mobile phone sounds

1: Original
2: Mobile phone quality
3: Processed from 2

Thanks to Ken (Mr. 史 宏杰)

Frequency response

Original
Mobile phone
Reconstructed

PESQ: Perceptual Evaluation of Speech Quality

- The computational time of the proposed method < 20ms
Epsilon filtering: distinguish details from noise by a nonlinear function (epsilon filter [Harashima et al. '82])

- Epsilon filtering: distinguish details from noise by a nonlinear function (epsilon filter [Harashima et al. '82])
- The number of taps of YY filter is very small (e.g. 16 taps)
- The nonlinear function (if-then function) is approximated by a continuous function.
- Don’t use MATLAB (interpreter) but a compiler.
  - We use Visual C++ with Open-CV
rational delays and ond synthesis

Pith h hit ed en e

\[ f(nh) \]
\[ f(rnh) \]

\[ \tau \]
\[ h \]
\[ r_h \]

ond synthesis a ple

- Original sound
  (A2, 110Hz)
- Shifted
  E3, 164.81Hz
  A3, 220Hz
  A4, 440Hz

a ple

- Music produced from a single tone (A2, 110Hz).
Conclusion

- Many applications
  - 4K TV, cell phones, super-resolution,
  - Optimal discretization (compression)
  - Sound synthesis, …
- In current SP literature, no explicit signal models are used
- Opportunities abound

Why not control?